

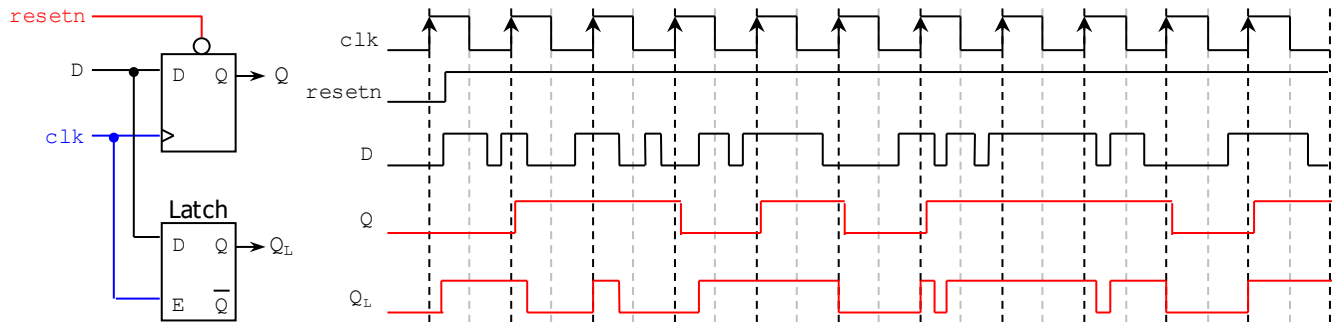
# Solutions - Final Exam

(December 12<sup>th</sup> @ 7:00 pm)

Presentation and clarity are very important! Show your procedure!

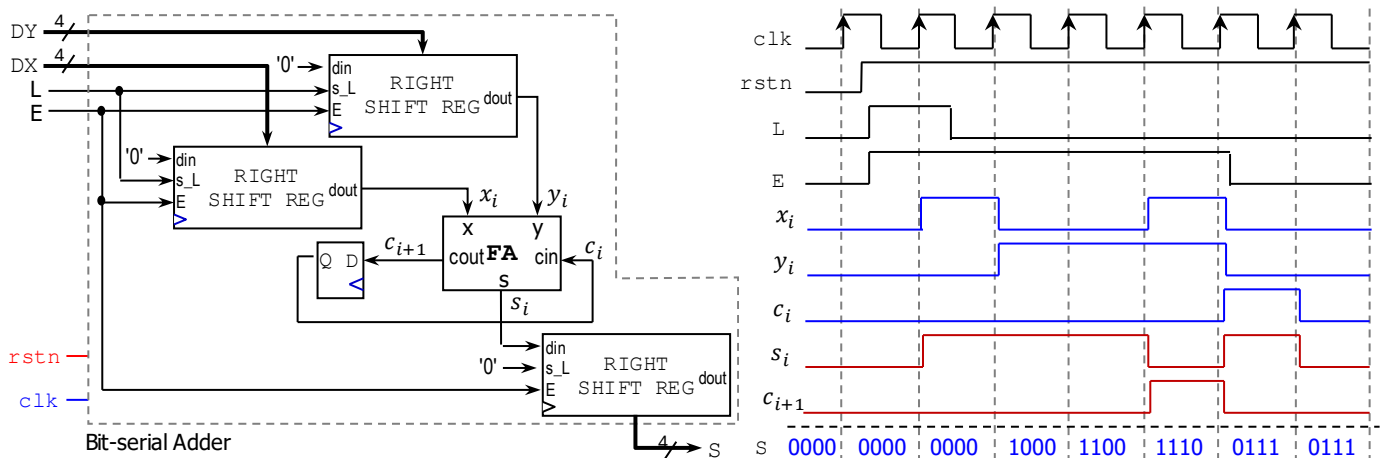
## PROBLEM 1 (18 PTS)

- Complete the timing diagram of the circuit shown below. (8 pts)



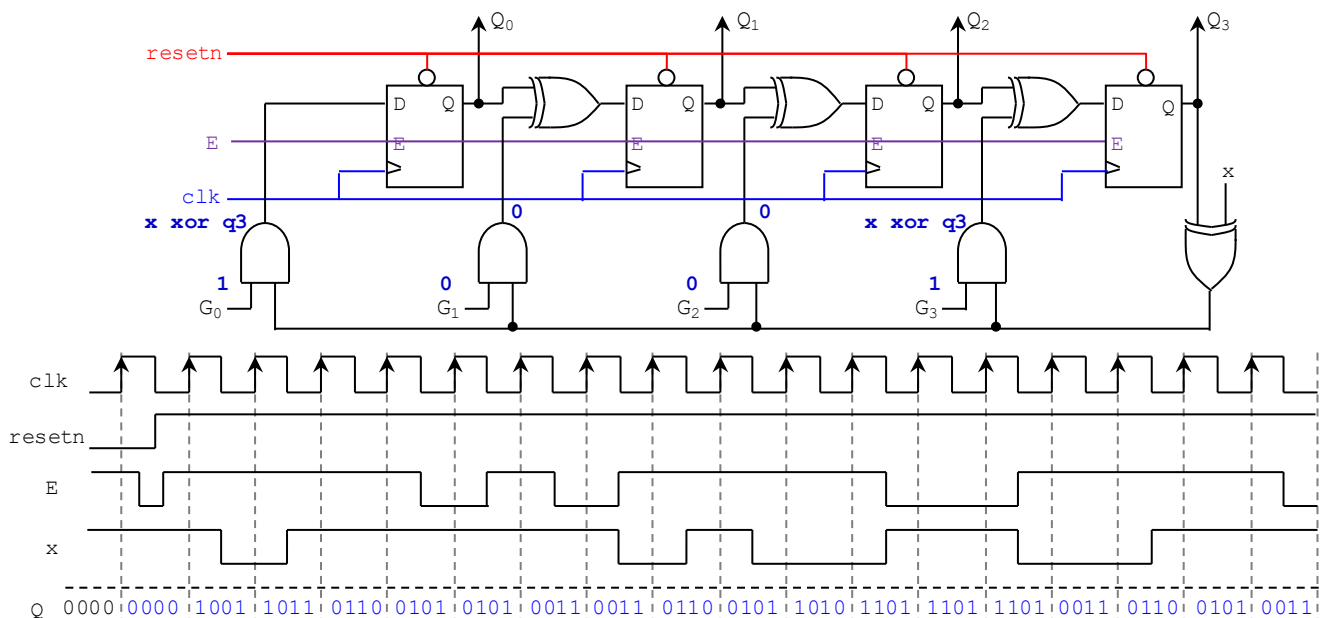
- Complete the timing diagram of the following bit-serial adder. DX=1001, DY=1110. (10 pts)

✓ This circuit includes three 4-bit parallel access shift registers, a flip flop, and a full adder.



## PROBLEM 2 (13 PTS)

- Complete the timing diagram of the following circuit.  $G = G_3G_2G_1G_0 = 1001$ ,  $Q = Q_3Q_2Q_1Q_0$



### PROBLEM 3 (13 PTS)

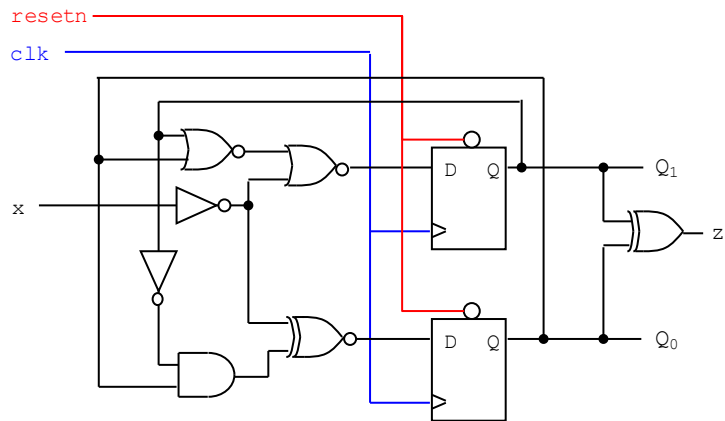
- Given the following circuit that represents a Finite State Machine, provide: (12 pts)
  - ✓ Excitation equations and output Boolean equation.
  - ✓ Excitation Table, State Table.
  - ✓ State Diagram (any representation).
- Is this a Mealy or a Moore Machine? (1 pt.)

$$Q_1(t+1) \leftarrow (Q_1(t) + Q_0(t))x$$

$$Q_0(t+1) \leftarrow (\overline{Q_1(t)}Q_0(t)) \oplus x$$

$$z = Q_1(t) \oplus Q_0(t)$$

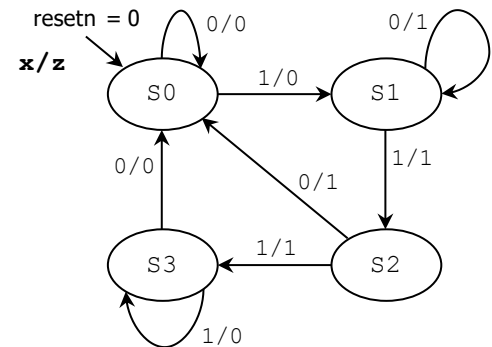
State assignment: S0: Q=00 S1: Q=01  
S2: Q=10 S3: Q=11



| PRESENT STATE |                                   |   | NEXTSTATE                           |   |  |
|---------------|-----------------------------------|---|-------------------------------------|---|--|
| x             | Q <sub>1</sub> Q <sub>0</sub> (t) |   | Q <sub>1</sub> Q <sub>0</sub> (t+1) | z |  |
| 0             | 0 0                               | 0 | 0 0                                 | 0 |  |
| 0             | 0 1                               | 1 | 0 1                                 | 1 |  |
| 0             | 1 0                               | 1 | 0 0                                 | 1 |  |
| 0             | 1 1                               | 0 | 0 0                                 | 0 |  |
| 1             | 0 0                               | 0 | 0 1                                 | 0 |  |
| 1             | 0 1                               | 1 | 1 0                                 | 1 |  |
| 1             | 1 0                               | 1 | 1 1                                 | 1 |  |
| 1             | 1 1                               | 0 | 1 1                                 | 0 |  |



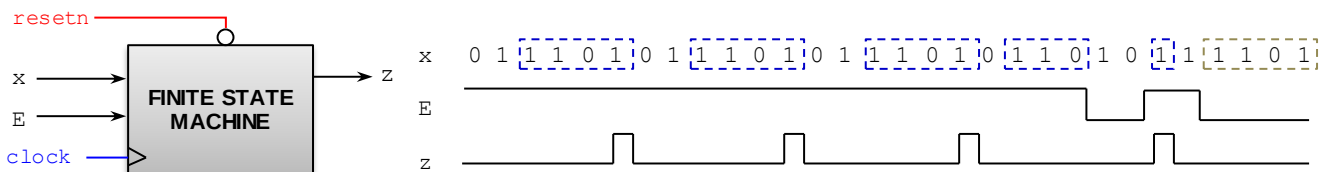
| PRESENT STATE |       |  | NEXT STATE |   |  |
|---------------|-------|--|------------|---|--|
| x             | STATE |  | STATE      | z |  |
| 0             | S0    |  | S0         | 0 |  |
| 0             | S1    |  | S1         | 1 |  |
| 0             | S2    |  | S0         | 1 |  |
| 0             | S3    |  | S0         | 0 |  |
| 1             | S0    |  | S1         | 0 |  |
| 1             | S1    |  | S2         | 1 |  |
| 1             | S2    |  | S3         | 1 |  |
| 1             | S3    |  | S3         | 0 |  |



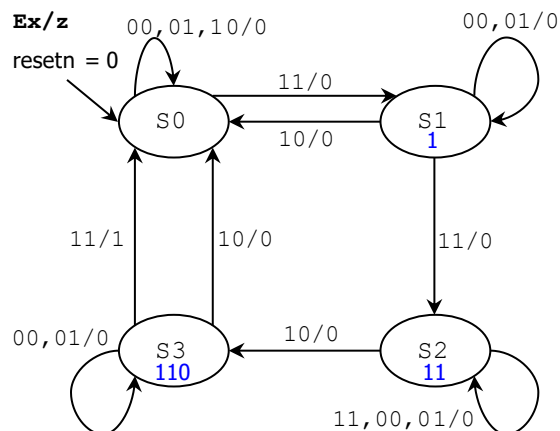
This a Moore FSM.

### PROBLEM 4 (16 PTS)

- Sequence detector: The machine generates  $z = 1$  when it detects the sequence 1101. Once the sequence is detected, the circuit looks for a new sequence.
- The signal E is an input enable: It validates the input  $x$ , i.e., if  $E=1$ ,  $x$  is valid, otherwise  $x$  is not valid.



- Draw the State Diagram (any representation) of this circuit with inputs E and  $x$  and output  $z$ . (7 pts.)
- Complete the State Table and the Excitation Table. (8 pts.)
- Is this a Mealy or a Moore machine? Why?



State assignment: S0: Q=00 S1: Q=01  
S2: Q=10 S3: Q=11

This a Mealy FSM. The output  $z$  depends on the inputs and the present state.

| PRESENT STATE |   |       | NEXT STATE |   |  |
|---------------|---|-------|------------|---|--|
| E             | x | STATE | STATE      | z |  |
| 0             | 0 | S0    | S0         | 0 |  |
| 0             | 0 | S1    | S1         | 0 |  |
| 0             | 0 | S2    | S2         | 0 |  |
| 0             | 0 | S3    | S3         | 0 |  |
| 0             | 1 | S0    | S0         | 0 |  |
| 0             | 1 | S1    | S1         | 0 |  |
| 0             | 1 | S2    | S2         | 0 |  |
| 0             | 1 | S3    | S3         | 0 |  |
| 1             | 0 | S0    | S0         | 0 |  |
| 1             | 0 | S1    | S0         | 0 |  |
| 1             | 0 | S2    | S3         | 0 |  |
| 1             | 0 | S3    | S0         | 0 |  |
| 1             | 1 | S0    | S1         | 0 |  |
| 1             | 1 | S1    | S2         | 0 |  |
| 1             | 1 | S2    | S2         | 0 |  |
| 1             | 1 | S3    | S0         | 1 |  |



| PRESENT STATE |   |                                   | NEXTSTATE                           |   |  |
|---------------|---|-----------------------------------|-------------------------------------|---|--|
| E             | x | Q <sub>1</sub> Q <sub>0</sub> (t) | Q <sub>1</sub> Q <sub>0</sub> (t+1) | z |  |
| 0             | 0 | 0 0                               | 0 0                                 | 0 |  |
| 0             | 0 | 0 1                               | 0 1                                 | 0 |  |
| 0             | 0 | 1 0                               | 1 0                                 | 0 |  |
| 0             | 0 | 1 1                               | 1 1                                 | 0 |  |
| 0             | 1 | 0 0                               | 0 0                                 | 0 |  |
| 0             | 1 | 0 1                               | 0 1                                 | 0 |  |
| 0             | 1 | 1 0                               | 1 0                                 | 0 |  |
| 0             | 1 | 1 1                               | 1 1                                 | 0 |  |
| 1             | 0 | 0 0                               | 0 0                                 | 0 |  |
| 1             | 0 | 0 1                               | 0 0                                 | 0 |  |
| 1             | 0 | 1 0                               | 1 1                                 | 0 |  |
| 1             | 0 | 1 1                               | 0 0                                 | 0 |  |
| 1             | 1 | 0 0                               | 0 1                                 | 0 |  |
| 1             | 1 | 0 1                               | 1 0                                 | 0 |  |
| 1             | 1 | 1 0                               | 1 0                                 | 0 |  |
| 1             | 1 | 1 1                               | 0 0                                 | 1 |  |

## PROBLEM 5 (18 PTS)

- "Counting 0's" Circuit: It counts the number of bits in register  $A$  that has the value of '0'.
  - ✓ Example: for  $n = 8$ : if  $A = 00110010$ , then  $C = 0101$ .
  - ✓ The behavior (on the clock tick) of the generic components is as follows:

$m$ -bit counter (modulo- $n+1$ ): If  $E=0$ , the count stays.

```

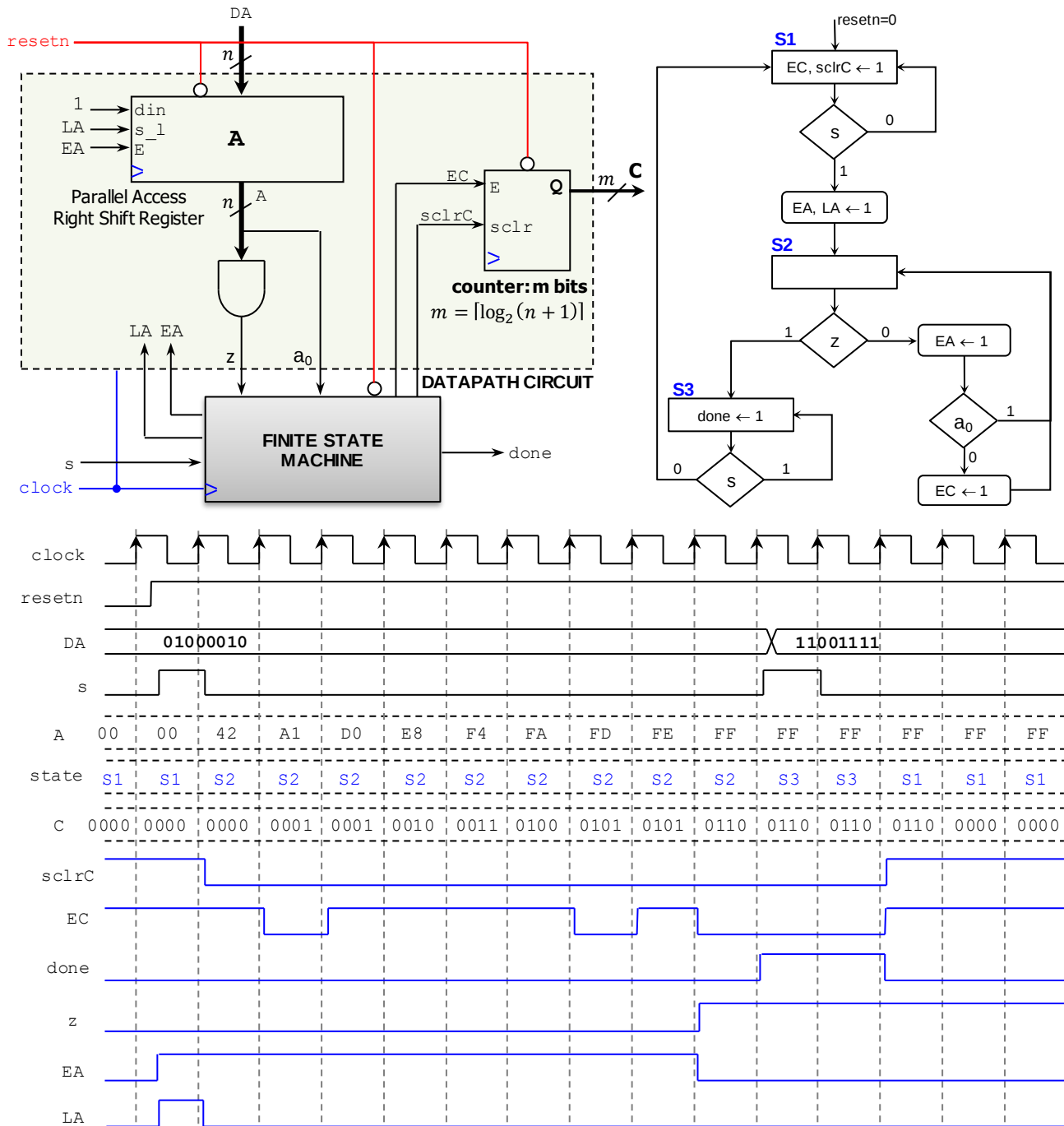
if E = 1 then
  if sclr = 1 then
    Q ← 0
  else
    Q ← Q+1
  end if;
end if;
    
```

$n$ -bit Parallel access shift register: If  $E=0$ , the output is kept.

```

if E = 1 then
  if s_l = '1' then
    Q ← D
  else
    Q ← shift in 'din' (to the right)
  end if;
end if;
    
```

- Complete the timing diagram where  $n = 8, m = 4$ .  $A$  is represented in hexadecimal format, while  $C$  is in binary format.

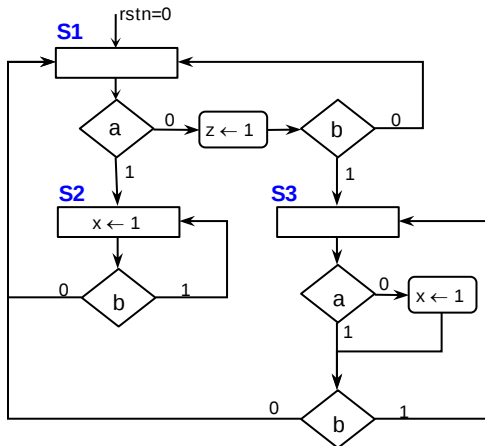


## PROBLEM 6 (22 PTS)

- Draw the State Diagram (in ASM form) of the FSM whose VHDL description is shown below. (6 pts)
- Provide the State Table and the Excitation Table. Is it a Mealy or a Moore FSM? (6 pts).
- Provide the excitation equations and the Boolean output equations (simplify your circuit: K-maps or Quine-McCluskey).
- Sketch the circuit. (3 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
port ( clk, rstn: in std_logic;
      a, b: in std_logic;
      x, z: out std_logic);
end circ;
```



```
architecture behavioral of circ is
type state is (S1, S2, S3);
signal y: state;
begin
Transitions: process (rstn, clk, a, b)
begin
if rstn = '0' then y <= S1;
elsif (clk'event and clk = '1') then
case y is
when S1 =>
if a = '1' then
y <= S2;
else
if b = '1' then y <= S3; else y <= S1; end if;
end if;
when S2 =>
if b = '1' then y <= S2; else y <= S1; end if;
when S3 =>
if b = '1' then y <= S3; else y <= S1; end if;
end case;
end if;
end process;

Outputs: process (y, a, b)
begin
x <= '0'; z <= '0';
case y is
when S1 => if a = '0' then z <= '1'; end if;
when S2 => x <= '1';
when S3 => if a = '0' then x <= '1'; end if;
end case;
end process;
end behavioral;
```

### State Table and Excitation Table:

| PRESENT STATE |   | NEXT STATE |       | x z |   |
|---------------|---|------------|-------|-----|---|
| a             | b | STATE      | STATE | x   | z |
| 0             | 0 | S1         | S1    | 0   | 1 |
| 0             | 0 | S2         | S1    | 1   | 0 |
| 0             | 0 | S3         | S1    | 1   | 0 |
| 0             | 1 | S1         | S3    | 0   | 1 |
| 0             | 1 | S2         | S2    | 1   | 0 |
| 0             | 1 | S3         | S3    | 1   | 0 |
| 1             | 0 | S1         | S2    | 0   | 0 |
| 1             | 0 | S2         | S1    | 1   | 0 |
| 1             | 0 | S3         | S1    | 0   | 0 |
| 1             | 1 | S1         | S2    | 0   | 0 |
| 1             | 1 | S2         | S2    | 1   | 0 |
| 1             | 1 | S3         | S3    | 0   | 0 |

State assignment:

S1: Q=00

S2: Q=01

S3: Q=10

| PRESENT STATE |   |                                   | NEXTSTATE                           |   |   |
|---------------|---|-----------------------------------|-------------------------------------|---|---|
| a             | b | Q <sub>1</sub> Q <sub>0</sub> (t) | Q <sub>1</sub> Q <sub>0</sub> (t+1) | x | z |
| 0             | 0 | 0 0                               | 0 0                                 | 0 | 1 |
| 0             | 0 | 0 1                               | 0 0                                 | 1 | 0 |
| 0             | 0 | 1 0                               | 0 0                                 | 1 | 0 |
| 0             | 0 | 1 1                               | X X                                 | X | X |
| 0             | 1 | 0 0                               | 1 0                                 | 0 | 1 |
| 0             | 1 | 0 1                               | 0 1                                 | 1 | 0 |
| 0             | 1 | 1 0                               | 1 0                                 | 1 | 0 |
| 0             | 1 | 1 1                               | X X                                 | X | X |
| 1             | 0 | 0 0                               | 0 1                                 | 0 | 0 |
| 1             | 0 | 0 1                               | 0 0                                 | 1 | 0 |
| 1             | 0 | 1 0                               | 0 0                                 | 0 | 0 |
| 1             | 0 | 1 1                               | X X                                 | X | X |
| 1             | 1 | 0 0                               | 0 1                                 | 0 | 0 |
| 1             | 1 | 0 1                               | 0 1                                 | 1 | 0 |
| 1             | 1 | 1 0                               | 1 0                                 | 0 | 0 |
| 1             | 1 | 1 1                               | X X                                 | X | X |

This a Mealy FSM. The outputs  $x$  and  $z$  depend on the input as well as on the present state.

- Minimization, Excitation equations (including Boolean output equations), and circuit implementation:

$$Q_1(t+1) \leftarrow bQ_1(t) + \overline{a}b\overline{Q_0(t)}$$

$$Q_0(t+1) \leftarrow a\overline{Q_1(t)} \cdot \overline{Q_0(t)} + bQ_0(t)$$

$$x = Q_0(t) + \overline{a}Q_1(t)$$

$$z = \overline{a}Q_1(t) \cdot \overline{Q_0(t)}$$

$$Q_1(t+1)$$

|          |    |    |    |    |    |
|----------|----|----|----|----|----|
|          | ab | 00 | 01 | 11 | 10 |
| $Q_1Q_0$ | 00 | 0  | 1  | 0  | 0  |
|          | 01 | 0  | 0  | 0  | 0  |
|          | 11 | X  | X  | X  | X  |
|          | 10 | 0  | 1  | 1  | 0  |

$$Q_0(t+1)$$

|          |    |    |    |    |    |
|----------|----|----|----|----|----|
|          | ab | 00 | 01 | 11 | 10 |
| $Q_1Q_0$ | 00 | 0  | 0  | 1  | 1  |
|          | 01 | 0  | 1  | 1  | 0  |
|          | 11 | X  | X  | X  | X  |
|          | 10 | 0  | 0  | 0  | 0  |

$$x$$

|          |    |    |    |    |    |
|----------|----|----|----|----|----|
|          | ab | 00 | 01 | 11 | 10 |
| $Q_1Q_0$ | 00 | 0  | 0  | 0  | 0  |
|          | 01 | 1  | 1  | 1  | 1  |
|          | 11 | X  | X  | X  | X  |
|          | 10 | 1  | 1  | 0  | 0  |

$$z$$

|          |    |    |    |    |    |
|----------|----|----|----|----|----|
|          | ab | 00 | 01 | 11 | 10 |
| $Q_1Q_0$ | 00 | 1  | 1  | 0  | 0  |
|          | 01 | 0  | 0  | 0  | 0  |
|          | 11 | X  | X  | X  | X  |
|          | 10 | 0  | 0  | 0  | 0  |

